and, then, eliminating the variable Q_n, we can finally obtain the equations of motion of system (4.1) in the from (2.1).

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THE APPLICATION OF ASYMPTOTIC METHODS TO CERTAIN STOCHASTIC PROBLEMS OF THE DYNAMICS OF VIBROPERCUSSIVE SYSTEMS*

A.S. KOVALEVA

Motion of certain vibropercussive systems acted upon by a random, non-white noise perturbation is studied, using the limit theorems of the convergence of the solutions of stochastic differential equations to a diffusion process. The results, first obtained in /1, 2/ for smooth systems, are generalized to include systems with discontinuous and impulsive right-hand sides /3–5/ by approximating the discontinuous functions by a converging sequence of smooth functions. An analogous approach is described for vibropercussive systems, and regions of stability of the perturbed motion are constructed.

Analytic expressions describing the probability density and dispersions of velocity and coordinates are well known /6, 7/ in the case of linear systems excited by white noise, under elastic impact. The method of non-smooth transformations /8/ is used for more complex systems to construct the FPK equations characterizing the distribution of the energy of the oscillations /9, 10/. Basic results are also obtained for systems excited by white noise.

1. Consider a quasiconservative, vibropercussive system. The equation of motion and condition of impact against a one-sided stop have the form

$$x^{\cdot\cdot} + \Omega^2 x = \varepsilon g(t, x, x^{\cdot}, \varepsilon) \tag{1.1}$$

$$x = \Delta,\ x_+^{\cdot} = -Rx_-^{\cdot},\ R = 1 - \varepsilon^2 r,\ r = \text{const} = O(1) \tag{1.2}$$

Here Δ is the size of the gap ($\Delta > 0$) or displacement ($\Delta < 0$), $x_-^{\cdot}$ and $x_+^{\cdot}$ denote the velocities before and after the impact and ε is a small parameter. The piecewise-continuous function g characterizes the additional non-conservative terms and represents, for fixed x

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and $x^{\cdot}$, a measurable random process.

When $\varepsilon = 0$, the generating conservative system has the following general integral /7/:

$$x(t) = -J(\omega)\chi(\psi, \omega), \quad \omega = 2\pi/T \tag{1.3}$$

$$\chi(\psi, \omega) = T^{-1} \sum_{k=-\infty}^{\infty} (\Omega^2 - k^2\omega^2)^{-1} e^{ik\psi}, \qquad \psi = \omega(J)(t - t_0)$$

Here $\chi(\psi, \omega)$ is the periodic Green's function /11/, T is the period between collisions, J is the impact momentum and t_0 is the impact phase. The function $\chi(\psi, \omega)$ is continuous and its first derivative becomes discontinuous at $\psi = 2\pi k$, $k = 0, \pm 1, \ldots$. The relation between the frequency of motion and the momentum is described by the following condition of impact:

$$\Delta \neq 0{:}\ \ x(t_0) = \Delta, \ \ J(\omega) = \Delta/\chi(0, \omega) = -2\Omega\,\mathrm{tg}\,(\pi\Omega/\omega) \tag{1.4}$$
$$\Delta = 0{:}\ \ \omega = 2\Omega$$

Assuming that the perturbations are small and the impact nearly elastic, we can presuppose that the character of the motion is preserved in the quasiconservative vibropercussive system and the mode of motion is almost a single T-periodic impact. Then the perturbed motion can be analysed using the methods employed to analyse systems which are almost conservative.

We shall use the procedure given in /2/ to reduce system (1.1) to standard form. Introducing the new momentum-phase variables

$$x = -J\chi(\psi, \omega), \ \ x^{\cdot} = -J\omega\chi_{\psi}(\psi, \omega) \tag{1.5}$$

we obtain, after transformations, equations analogous to those given in /12/

$$J^{\cdot} = -4\varepsilon g(t, -J\chi, -J\omega\chi_{\psi}, \varepsilon)\,\omega\chi_{\psi} \tag{1.6}$$
$$\psi^{\cdot} = \omega(J)\,[1 + 4\varepsilon J^{-1} g(t, -J\chi, -J\omega\chi_{\psi}, \varepsilon)\,(J\chi)_J]$$

The relation $\omega(J)$ is given by condition (1.4). The derivative $(J\chi)_J$ is computed taking the relation connecting χ and $\omega(J)$ into account. The right-hand sides of (1.6) are 2π-periodic in ψ. After substituting (1.5) into (1.4) the conditions of impact are transformed into the conditions of discontinuity of the momentum: at the instant of impact when $\psi = 2\pi k$ $(k = 0, \pm 1, \pm 2, \ldots)$, $J_+ = RJ_-$, i.e. when $R = 1 - \varepsilon^2 r$

$$J_+ - J_- = -\varepsilon^2 r J_-, \ \ \psi = 2\pi k \tag{1.7}$$

The condition of impact (1.7) can be inserted into the first equation of (1.5) with help of the δ-function /13/. Taking into account the relation $\psi(t)$ we obtain, to terms $O(\varepsilon^3)$,

$$J^{\cdot} = -4\varepsilon g\omega\chi_{\psi} - \varepsilon^2 r \sum_{(k)} \omega(J_k)\, J_k \delta(\psi - 2\pi k) \tag{1.8}$$
$$\psi^{\cdot} = \omega(J)\,[1 + 4\varepsilon J^{-1} g\,(J\chi)_J]$$
$$g = g(t, -J\chi, -J\omega\chi_{\psi}, \varepsilon), \ J_k = J_-|_{\psi = 2\pi k}$$

The non-isochronous character of the system, i.e. the dependence of the frequency ω on the momentum J, leads to computational complications. Henceforth, we shall assume that the gap is small $\Delta = \varepsilon^2\Delta_1$. This will simplify the computations considerably, without affecting the qualitative representations concerning the form of the solution, since the generating system will become isochronous $\omega = \omega_0 = 2\Omega$. Let us introduce the substitution

$$x = x_1 + \varepsilon^2\Delta_1; \tag{1.9}$$
$$x_1 = -J\chi^{\circ}(t - \varphi), \ \ x_1^{\cdot} = -J\chi_t^{\circ}(t - \varphi)$$

where $\chi^{\circ}(t) = \chi(2\Omega t, 2\Omega)$, and when $0 \leqslant t \leqslant \pi/\Omega$ /11/, we have

$$\chi^{\circ}(t) = (2\Omega)^{-1}\sin\Omega t, \ \ \chi_t^{\circ} = {}^1/_2\cos\Omega t \tag{1.10}$$

The substitution (1.9) reduces the initial equation to its standard form

$$J^{\cdot} = -4\varepsilon\,[g - \varepsilon\Delta_1\Omega^2]\,\chi_t^{\circ}(t - \varphi) - \varepsilon^2 r \sum_{(k)} J_k \delta(t - \varphi - kT), \ \ T = \pi/\Omega \tag{1.11}$$
$$\varphi^{\cdot} = -4\varepsilon J^{-1}\,[g - \varepsilon\Delta_1\Omega^2]\,\chi^{\circ}(t - \varphi)$$

The right hand sides of (1.11) are $T = \pi/\Omega$-periodic in t. The equations obtained can be analysed using the results obtained in /3–5/.

2. Let the system be isochronous and $g = g_1(t, x, x^{\cdot}) + \varepsilon g_2(t, x, x^{\cdot})$, so that

$$J^{\cdot} = -\varepsilon^2 r \sum_{(k)} J_k \delta(t - \varphi - kT) + \varepsilon G_{11}(t, J, \varphi) + \varepsilon^2 G_{21}(t, J, \varphi) \tag{2.1}$$
$$\varphi^{\cdot} = \varepsilon G_{12}(t, J, \varphi) + \varepsilon^2 G_{22}(t, J, \varphi)$$

$$G_{j1} = -4g_j(t, -J\chi^{\circ}(t - \varphi), -J\chi_t^{\circ}(t - \varphi))\,\chi_t^{\circ}(t - \varphi) \tag{2.2}$$
$$G_{j2} = -4J^{-1} g_j(t, -J\chi^{\circ}(t - \varphi), -J\chi_t^{\circ}(t - \varphi))\,\chi^{\circ}(t - \varphi)$$
$$j = 1, 2$$

Let the functions g_1 and g_2 be continuous together with their derivatives in x and $x^{\cdot}$ and represent, for fixed x and $x^{\cdot}$, a measurable random process satisfying the condition of strong mixing /1/, with $Mg_1 = 0$.

Let us put $z = (J, \varphi)$ and introduce the moment characteristics

$$B_j(t, s, z) = M \sum_{r=1}^{2} \frac{\partial G_{1j}(t, z)}{\partial z_r} G_{1r}(s, z) \tag{2.3}$$
$$a_{ij}(t, s, z) = MG_{1i}(t, z) G_{1j}(s, z), \quad A = \{a_{ij}\}, \quad i, j = 1, 2$$

Let the conditions given above for the functions g_1, g_2 all hold, and let the following limits exist uniformly in z and t_0:

$$\lim_{\theta\to\infty} \frac{1}{\theta} \int_{t_0}^{\theta+t_0} MG_{2j}(t, z)\,dt = \bar{G}_{2j}(z) \tag{2.4}$$
$$\lim_{\theta\to\infty} \frac{1}{\theta} \int_{t_0}^{\theta+t_0} dt \int_{t_0-\theta}^{t} B_j(t, s, z) = \bar{B}_j(z), \qquad j = 1, 2$$
$$\lim_{\theta\to\infty} \frac{1}{\theta} \int_{t_0}^{\theta+t_0} dt \int_{t_0}^{\theta+t_0} A(t, s, z) = \bar{A}(z)$$

Then /1–5/ the solution of (2.1) converges weakly to the solution of the system of Ito equations

$$dJ^\circ = [\bar{G}_{21}(J^\circ, \varphi^\circ) + \bar{B}_1(J^\circ, \varphi^\circ) - rT^{-1}J^\circ]\,d\tau + \tag{2.5}$$
$$\sigma_{11}(J^\circ, \varphi^\circ)\,dw_1 + \sigma_{12}(J^\circ, \varphi^\circ)\,dw_2$$
$$d\varphi^\circ = [\bar{G}_{22}(J^\circ, \varphi^\circ) + \bar{B}_2(J^\circ, \varphi^\circ)]\,d\tau + \sigma_{21}(J^\circ, \varphi^\circ)\,dw_1 -$$
$$\sigma_{22}(J^\circ, \varphi^\circ)\,dw_2$$
$$\sigma\sigma' = \bar{A}, \quad \tau = \varepsilon^2 t, \quad 0 \leqslant \tau \leqslant \tau_0$$

where $w = \{w_1, w_2\}$ is a two-dimensional standard Wiener process.

If $g_j(t, x, x^\cdot) = g_j(x, x^\cdot)\,\xi_j(t)$, where $\xi_j(t)$ are stationary random processes with fairly rapidly diminishing correlation functions, we can confirm that $\bar{B} = \bar{B}(J)$, $\bar{A} = \bar{A}(J)$, i.e. the coefficicnts of (2.5) are independent of φ and the first equation of (2.5) can be separated.

3. Relations (2.1)–(2.5) are of a general character. We shall use them to solve some problems of the dynamics of vibropercussive systems.

1°. Consider a system with parametric perturbations, linear in the intervals between impacts. Its motion is described by the equation

$$x^{\cdot\cdot} + \Omega^2 (1 + \varepsilon\xi(t))\,x + 2\varepsilon^2 b x^\cdot = 0 \tag{3.1}$$

and conditions of impact (1.2). We shall assume that the gap is small $\Delta = \varepsilon^2\Delta_1$. Then the substitution (1.9) will transform (3.1) to the form (1.11).

It is important in most cases to know the behaviour of the mean square value of the variable J. Introducing the new variable $y = J^2$, we transform system (1.1) to the form

$$y^\cdot = -8\varepsilon y\Omega^2\xi(t)\,\chi^\circ(t-\varphi)\,\chi_t^\circ(t-\varphi) - 8\varepsilon^2 \{[2by\chi_t^\circ(t-\varphi) - \tag{3.2}$$
$$\Omega^2\Delta_1 y^{1/2}]\,\chi_t^\circ(t-\varphi) + \frac{r}{4}\sum_{(k)} y_k\delta(t-\varphi-kT)\}$$
$$\varphi^\cdot = -4\varepsilon\Omega^2\xi(t)\,[\chi^\circ(t-\varphi)]^2 - 4\varepsilon^2\,[2b\chi_t^\circ(t-\varphi) -$$
$$\Omega^2\Delta_1 y^{-1/2}]\,\chi^\circ(t-\varphi)$$

We write, in accordance with (2.1),

$$G_{11} = -8y\Omega^2\xi(t)\,\zeta_1(t-\varphi) \tag{3.3}$$
$$G_{12} = -4\Omega^2\xi(t)\,\zeta_2(t-\varphi)$$
$$G_{21} = -8\,[2by\zeta_3(t-\varphi) - \Omega^2\Delta_1 y^{1/2}\chi_t^\circ(t-\varphi)]$$
$$G_{22} = -4\,[2b\zeta_1(t-\varphi) - \Omega^2\Delta_1 y^{-1/2}\chi^\circ(t-\varphi)]$$
$$\zeta_1(t) = \chi^\circ(t)\,\chi_t^\circ(t), \quad \zeta_2(t) = [\chi^\circ(t)]^2, \quad \zeta_3(t) = [\chi_t^\circ(t)]^2$$

By virtue of (1.10) the functions ζ_j $(j = 1, 2, 3)$ are continuous and

$$\zeta_1(t) = (8\Omega)^{-1}\sin 2\Omega t, \quad \zeta_2(t) = (8\Omega^2)^{-1}\,(1-\cos 2\Omega t)$$
$$\zeta_3(t) = 1/8\,(1+\cos 2\Omega t), \quad 0 \leqslant t < \infty$$

Let $\xi(t)$ be a stationary random process with a fairly rapidly decreasing correlation function, and spectral density $S(\lambda)$. Then, computing the coefficients of equations (2.4), we obtain

$$\bar{G}_{21} = -2by^\circ, \quad \bar{B}_1 = 1/2\,\Omega^2 S(2\Omega)\,y^\circ$$
$$a_{11} = 1/2\,\Omega^2 (y^\circ)^2 S(2\Omega), \quad a_{12} = 0$$

Consequently the following linear Ito equation corresponds to the process y°:

$$dy^\circ = \beta y^\circ d\tau + a_{11}^{1/2} dw \tag{3.4}$$
$$\beta = 1/2\,\Omega^2 S(2\Omega) - 2\,(b + r\pi^{-1}\Omega)$$

The condition of stability in the mean square follows from (3.4) at once: $\beta < 0$. A weaker condition of probability stability was obtained in /9/ for systems perturbed by white noise.

Note that when $r = 0$ we obtain the well-known condition of stability of a system without a stop; if $\xi(t) = 0$ and $b = -b_1 < 0$, we obtain the conditions for the quenching unstable oscillations in the case of inelastic impact

$$r < \pi\Omega^{-1} \mid b \mid \tag{3.5}$$

which agrees with that established in /9/.

2°. The motion of a vibropercussive system excited by a random force is described by the equation

$$x^{\cdot\cdot} + 2\varepsilon^2 bx^{\cdot} + \Omega^2 x = \varepsilon\xi(t) \tag{3.6}$$

and the conditions of impact (1.2); $\xi(t)$ is a stationary random process with a fairly rapidly decreasing correlation function, and spectral density $S(\lambda)$. Assuming that the gap is small $\Delta = \varepsilon^2\Delta_1$ and putting (1.9) $J = y^{1/2}$, we reduce (3.6) to the form

$$y^{\cdot} = -2\varepsilon^2 r \sum_{(k)} y_k \delta(t - \varphi - kT) + \varepsilon G_{11} + \varepsilon^2 G_{21}$$

$$\varphi^{\cdot} = \varepsilon G_{12} + \varepsilon^2 G_{22}$$

$$G_{11} = -8y^{1/2}\xi(t)\chi_t{}^{\circ}(t - \varphi), \quad G_{12} = -4y^{-1/2}\xi(t)\chi^{\circ}(t - \varphi)$$

where the functions G_{21}, G_{22} are identical with those computed in Sect.1.

The diffusion equation for the limit process y° has the form

$$dy^{\circ} = (\bar{G}_{21} + \bar{B}_1 - 2r\pi^{-1}\Omega y^{\circ})\,d\tau + \sigma_{11}(y^{\circ})\,dw$$

Here, as before, we have $\bar{G}_{21} = -2by^{\circ}$ and the quantities $\bar{B}_1, \sigma_{11}$ computed from (2.4), have the form

$$\bar{B}_1 = \frac{64}{\pi^2} \sum_{k=1}^{\infty} \frac{4k^2}{(4k^2-1)^2} S(k\omega), \quad \sigma_{11}{}^2 = a_{11} = \bar{B}_1 y_0 \tag{3.7}$$

If $S(\lambda)$ is a rational fractional function and the correlation time $\xi(t)$ of the process is much shorter than $T = \pi\Omega^{-1}$, then summing the series (3.7) /11/ and remembering that $\omega = 2\Omega$, we obtain

$$\bar{B}_1 \approx 4S(\Omega)$$

We obtain a linear equation for the moment $m = M(y^{\circ})^2$ with the following stationary solution:

$$\bar{m} = 2S(\Omega)\,[b + r\pi^{-1}\Omega]^{-1}$$

We see that the dissipation of energy on impact ($r \neq 0$) reduces the oscillation intensity in the case of a random, wide-band input and assists, when condition (3.5) holds, in quenching the unstable oscillations in the system without a stop.

Notes. 1°. The procedure given here remains valid for systems with extra degrees of freedom whose dynamics can be described by the equations.

$$x^{\cdot\cdot} + \Omega^2 x = \varepsilon g(t, \tau, x, x^{\cdot}, \varepsilon)$$

$$y^{\cdot} = \varepsilon Y(t, \tau, x, x^{\cdot}, \varepsilon), \quad y \in R_n, \quad \tau = \varepsilon^2 t$$

and the impact condition $x = \Delta, x_+^{\cdot} = -Rx_-^{\cdot}, R = 1 - \varepsilon^2 r$. Here the function Y satisfies all the requirement of Sect.2.

2°. The case of a two-sided stop is considered in the same manner. The function χ in the expression for the general integral of the conservative system (1.3) and in the substitution (1.5) is replaced here by the following periodic Green's function of second kind /11/:

$$\chi_{2T} = T^{-1} \sum_{k=-\infty}^{\infty} [-(2k-1)^2\omega^2 + \Omega^2]^{-1} e^{(2k-1)i\psi}$$

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ON THE PRECESSIONAL - SCREW MOTIONS OF A SOLID IMMERSED IN LIQUID*

V.N. RUBANOVSKII

The Kirchhoff-Klebsch problem on the inertial motion of a solid immersed in a liquid is considered. The precessional-screw motions of the solid consisting of two screw motions are investigated. The axis of one motion is fixed in space, and the other axis is fixed within the body. The necessary and sufficient kinematic conditions are given for the precessional-screw motions in differential and finite form. A method of finding such motions is given, their stability is studied and a geometrical interpretation of the body motions is presented.

1. Consider the problem of the inertial motion of a free solid bounded by a singly connected surface and containing multiconnected cavities completely filled with a perfect fluid in irrotational motion, in a perfect, homogeneous incompressible fluid unbounded in all directions. We will assume that the motion of the fluid outside the body caused by its motion is irrotational, and that the fluid is at rest at infinity.

The kinetic energy T of such a dynamic system can be written, apart from a constant determined by the periodic motion of the fluid within the body cavities, in the form /1/

$$T=\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}(a_{ij}P_iP_j+b_{ij}R_iR_j+2c_{ij}P_iR_j),\quad a_{ij}=a_{ji},\ b_{ij}=b_{ji}$$

where a_{ij}, b_{ij}, c_{ij} are constants defined for the given system, while $R_1\ R_2, R_3$ and P_1, P_2, P_3 are projections of the impuslive force $\mathbf{R}$ and impulsive couple $\mathbf{P}$ of the system on the axes of a rectangular $Ox_1x_2x_3$ coordinate system rigidly bound to the body, neglecting the cyclic motion of the fluid within the body cavities.

Denoting by u_i and Ω_i the projections of the translational velocity $\mathbf{u}$ and instantaneous angular velocity $\mathbf{\Omega}$ of the body on the x_i axes, we obtain for them the following expressions:

$$u_1=\partial T/\partial R_1,\qquad \Omega_1=\partial T/\partial P_1\qquad (123)\tag{1.1}$$

The equations of motion of a body in a fluid have the form /1, 2/

$$d\mathbf{R}/dt+\mathbf{\Omega}\times\mathbf{R}=0,\quad d\mathbf{P}/dt+\mathbf{\Omega}\times(\mathbf{P}+\mathbf{k})+\mathbf{u}\times\mathbf{R}=0\tag{1.2}$$

where $\mathbf{k}=(k_1, k_2, k_3)$ is the kinetic momentum vector of the cyclic motion of the fluid in the body cavities. Equations (1.2) admit of three first integrals

$$T=E=\text{const},\ \mathbf{R}^2=H^2=\text{const},\ (\mathbf{P}+\mathbf{k})\cdot\mathbf{R}=hH^2=\text{const}\tag{1.3}$$

Using the methods of screw calculus /3/ we introduce the impulsive $\mathbf{Q}=\mathbf{R}+\omega(\mathbf{P}+\mathbf{k})$ and kinematic $\mathbf{U}=\mathbf{\Omega}+\omega\mathbf{u}$ screw, where $\omega\ (\omega^2=0)$ is the Clifford number, and write (1.2) in the form of a single equation

$$d\mathbf{Q}/dt+\mathbf{U}\times\mathbf{Q}=0\tag{1.4}$$

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